Ge249_

COURSE OBJECTIVE

This course introduces the student to college level algebra. Students explore the major concepts of algebra including polynomials, quadratic equations, linear equations and functions.

ge249_ch1_1
OBJECTIVES
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Upon completion of this unit, you will be able to:

- Add and subtract positive and negative integers
- Multiply and divide positive and negative integers
- Solve signed fraction addition problems
- Solve signed fraction multiplication problems
- Evaluate expressions with exponents
- Apply substitution
- Apply the distributive property
- Combine like terms
- Solve one and two step equations with the Properties of Equality
- Solve inequalities
- Graph inequalities
- Solve word problems with linear equations

ge249_ch1_2

INTEGERS

ADDING & SUBTRACTING INTEGERS

First, let's review some rules about working with negatives.

If you have a positive and a negative number

1. Subtract
2. Take the sign of your largest number

Example: -10 + 2 = -8

Explanation: The ten is negative and the two is positive. Since the signs are different, the numbers are subtracted. The sign of the largest number, which is 10, is negative; therefore the answer is negative.

Example: 5 – 2 = 3

Explanation: The five is positive and the two is negative. Since the signs are different, the numbers are subtracted. The sign of the largest number, which is 5, is positive; therefore the answer is positive.

Practice

1. -6 + 2 = -4

2. 4 – 2 = 2

Answers

1. -4

The six is negative and the two is positive. Since the signs are different, the numbers are subtracted. The sign of the largest number, which is 6, is negative; therefore the answer is -4.

2. 2

The four is positive and the two is negative. Since the signs are different, the numbers are subtracted. The sign of the largest number, which is 4, is positive; therefore the answer is 2.

ge249_ch1_3

INTEGERS

MULTIPLYING INTEGERS

Note the rules below for multiplying integers.

Multiplying Rules

1. A positive times a positive equals a positive.

   8 x 3 = 24

2. A positive times a negative equals a negative.

   (8) x (-3) = -24

3. A negative times a negative equals a positive.

   (-8) x (-3) = 24

Practice

1. (-5) x (-3)

2. (-10) x (3)

3. 9 x 9

Answers

1. 15

Both numbers are negative; a negative times a negative equals a positive; therefore the answer is a positive 15.

2. -30

The 10 is negative and the 3 is positive. Since a negative times a positive equals a negative, the answer is -30.

3. 81

Both numbers are positive; a positive times a positive equals a positive; therefore the answer is a positive 81.

ge249_ch1_4

INTEGERS

DIVIDING INTEGERS

Note the rules below for dividing integers.

Dividing Rules

1. A positive divided by a positive equals a positive.

   15 ÷ 3 = 5

2. A positive divided by a negative equals a negative.

   (15) ÷ (-3) = - 5

3. A negative divided by a negative equals a positive.

   (-15) ÷ (-3) = 5

Practice

1. (-10) ÷ (-2)
2. (-12) ÷ (3)
3. 20 ÷ 10

Answers

1. 5
Both numbers are negative; a negative divided by a negative equals a positive; therefore the answer is a positive 5.

2. -4
The 12 is negative and the 3 is positive. When there is one negative number in a division problem, the answer is positive; therefore the answer is -4.

3. 2
Both numbers are positive, therefore the answer is 2.

ge249_ch1_5

FRACTIONS

SIGNED FRACTION ADDITION

Signed fraction addition includes adding fractions with negative numbers. Take a look at the problem below. Notice the 3 is negative, and it is located on the bottom of the fraction, or denominator.

$$\frac{2}{-3} + \frac{5}{7}$$

In order to combine these fractions, the negative sign must be moved from the denominator and placed into the numerator. This is done by giving the top number the negative sign.

$$\frac{-2}{3} + \frac{5}{7}$$

In order to add these two fractions together we must find the common denominator. To find the common denominator, we must find a number that 3 and 7 can go into equally. Since 3 and 7 will go into 21 evenly, we will use 21 as the common denominator.

Rewriting the problem vertically with the common denominator, we have

$$\frac{-2}{3} = \frac{-14}{21}$$

$+$

$$\frac{6}{7} = \frac{18}{21}$$

\---

$$\frac{4}{21}$$

The answer is $\frac{4}{21}$. The numerators combined were -14 + 18 which equals 4. The denominator is 21.

Practice

1. $\frac{3}{-10} + \frac{4}{5}$

2. $\frac{5}{6} + \frac{1}{-4}$

3. $\frac{7}{-8} + \frac{1}{2}$

Answers

1. $\frac{35}{50}$ or $\frac{7}{10}$

2. $\frac{7}{12}$

3. $-\frac{6}{16}$ or $-\frac{3}{58}$

ge249_ch1_6

FRACTIONS

SIGNED FRACTION MULTIPLICATION

Multiplying fractions with negative numbers includes following a few rules we have already covered.

Let's review the rules for multiplying with negative numbers.

Multiplying Rules

1. A positive times a positive equals a positive.

   8 x 3 = 24

2. A positive times a negative equals a negative.

   (8) x (-3) = -24

3. A negative times a negative equals a positive.

   (-8) x (-3) = 24

Using the rules above, let's solve this problem:

$$\frac{6}{-7} \text{ x } \frac{-3}{8}$$

To multiply a fraction, multiply the numerators, then the denominators.
Numerator: 6 x -3
Denominator: -7 x 8

$$\frac{6}{-7} \text{ X } \frac{-3}{8} = \frac{-18}{-56} = \frac{9}{28}$$

....

The multiplication rules are easy to apply if the problem only contains two numbers, but what if it contains more than two?

For example, what if we had a problem like below:

$$\frac{-3}{8} \; x \; \frac{2}{-5} \; x \; \frac{-1}{4}$$

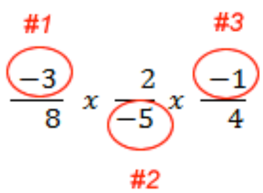
A simple way to find the sign of the answer is to count all of the negative numbers in the problem.

Then, follow this rule:

- If the amount of negatives is an **even** number, the answer will be **positive**. Even meaning 2, 4, 6, 8, 10, etc.

- If the amount of negatives is an **odd** number, the answer will be **negative**. Odd meaning 1, 3, 5, 7, 9, etc.

Revisiting this problem $\frac{-3}{8}\ x\ \frac{2}{-5}\ x\ \frac{-1}{4}$, let's count the negative numbers.



There are three negative numbers. Since 3 is an **odd** number, our answer will be **negative**.

Now that we know the answer will be negative, we can multiply our fraction:

$$\frac{-3}{8}\ x\ \frac{2}{-5}\ x\ \frac{-1}{4} = -\ \frac{6}{160} = -\ \frac{3}{80}$$

Practice

1. $\frac{8}{5}$ X $\frac{-2}{7}$ X $\frac{-1}{3}$

2. $\frac{-1}{12}$ X $\frac{-3}{2}$ X $\frac{-1}{-5}$

3. $\frac{-1}{-5}$ X $\frac{-1}{7}$ X $\frac{-1}{-4}$

Answers

1. $\frac{16}{105}$

2. $\frac{3}{120} = \frac{1}{40}$

3. $-\frac{1}{140}$

ge249_ch1_7

EXPONENTS

EVALUATING EXPONENTS

What are exponents? Exponents are numbers written in a superscript format, that indicate how many times a number, called the base number, should be multiplied times itself.

For example, in 6^2, indicates 6 x 6. The six is the base number, and the 2 is the exponent. It is read “six to the second power” or “six squared”,

Here are some examples of exponent problems:

$5^3 = 5 \times 5 \times 5 = 125$

$7^2 = 7 \times 7 = 49$

$2^6 = 2 \times 2 \times 2 \times 2 \times 2 \times 2 = 64$

$$\left(\frac{5}{6}\right)^2 = \left(\frac{5}{6}\right) \times \left(\frac{5}{6}\right) = \left(\frac{25}{36}\right)$$

$$\frac{2}{6}^3 = \frac{2 \times 2 \times 2}{6} = \frac{8}{6}$$

Now let’s look at some examples with exponents and negative numbers.

$(-5)^2 = -5 \times -5 = 25$

$(-2)^5 = -2 \times -2 \times -2 \times -2 \times -2 = -32$

$$\left(-\frac{4}{5}\right)^2 = \left(-\frac{4}{5}\right) \times \left(-\frac{4}{5}\right) = \frac{16}{25}$$

$$\frac{(-2)^2}{10} = \frac{(-2) \times (-2)}{10} = \frac{4}{10}$$

Practice

1. $\frac{(-4)^2}{3}$

2. [redacted]

3. $\left(-\frac{2}{4}\right)^3$

Answers

1. $\frac{16}{3}$

2. 343

3. $-\frac{8}{64}$

ge249_ch1_8

SUBSTITUTION

APPLYING SUBSTITION WITH A GIVEN VALUE

In Algebra, unknown values are indicated by a letter, called a variable. A value may be represented by any letter, but the most common variables are a, b, c, x, y, z.

In a + 10, a is the variable, or unknown value.

When substituting, the value will be given to substitution or “plug-in” for the variable.

For example, if we are given a + 10, and the value of a = 3, substitute 3 for a:

a + 10

3 + 10 Substituted 3 for a

13 Added 3 + 10

The answer would be 13.

Let’s try another:

$x^2 + 14$, when x = 4

$4^2 + 14$ Substituted 4 for x

16 + 14 $4^2 = 16$

30 Added 16 + 14

Now let’s look at an example with negative numbers.

$a^2 + a$, when x = -5

$(-5)^2 +(-5)$ Substituted -5 for x

(-5) x (-5) + (-5) $(-5)^2 = 25$

20 25-5= 20

Practice

1. b – 10 when b = 5
2. $a^3 + 5$ when a = 2

Answers

1. -5

2. 13

ge249_ch1_9

DISTRUBUTIVE PROPERTY

APPLYING THE DISTRIBUTIVE PROPERTY

When applying the distributive property, the term on the outside of parenthesis is multiplied by each part within the parenthesis. It is called the distributive property because the number is being distributed to each part.

Example:

4 (x+10)

4x + 40

Explanation:

4 (x + 10) Multiply 4 times x

4x

4 (x + 10) Multiply 4 times 10

40

Answer: 4x + 40

Example:

-20 (2a – 3)

-40a + 60

Explanation:

-20 (2a -3) Multiply -20 times 2a

-40a

-20 (2a -3) Multiply -20 times -3

60

Answer: -40a + 60

Practice

1. $4(x + 20)$

2. $-5(2a - 8)$

3. $10(4 - x)$

Answers

1. $4x + 80$

2. $-10a + 40$

3. $40 - 10x$

ge249_ch1_9

LIKE TERMS

COMBINING LIKE TERMS

When variables are included in a problem, only like terms can be combined.

If we have 3 + 5, the 3 and 5 are added to equal 8.

What if we add a variable to this problem?

3 + 5 + a

Numbers and variables cannot be added together, therefore

3 + 5 + a = 8 + a

Now let's look at variables are that are part of a number. The numbers before the variables are called **coefficients**. In the example below, the 2 and 5 are coefficients.

2a + **5**a

When a variable is part of a number, add the coefficients, and keep the variable.

2a + 5a = 7a

Since the 2 and 5 both have a's as variables, the 2 and 5 have **like terms.**

We can also have multiple variables:

3a + 2a + 4b

The 3 and 2 both have a's, or like terms, so 3a + 2a can be combined; the 4b must remain as 4b:

5a + 4b

In Algebra, if there is no coefficient indicated, then it is understood that it is a 1. For example:

1x is written as x
1y is written as y
1a is written as a

Evaluate: x + 2x
The answer is 3x as 1 + 2 = 3.

Evaluate: 5a – a

The answer is 4a as 5 - 1 = 4.

Practice

1. $9y - y$

2. $6a + 5a - 3b$

3. $7a + 20a$

Answers

1. $8y$

2. $11a - 3b$

3. $27a$

ge249_ch1_10

PROPERTIES OF EQUALITY

ADDITION PROPERTY OF EQUALITY

The next four lessons cover how to solve an equation with the properties of equality. An equation is signified by an equal sign. There are four properties of equality; these properties state that in order to solve an equation, the goal is to eliminate all numbers that are on the same side of the equal sign as the variable. To eliminate a number, we must either add, subtract, multiply or divide. Whatever is done to one side of the equation, must be done to the other,

This example will demonstrate the **addition** property of equality.

In the equation below,

x - 4 = 10

x is the variable, or the unknown value. We must use the Addition Property of Equality to find the value of x.

In x + 4 = 10, we must eliminate the 4.

$$\begin{array}{rcl} x - 4 & = & 10 \\ +4 & & +4 \\ \hline x & = & 14 \end{array}$$

Using the property of equality, we can eliminate the 4 by adding 4. Whatever is done to one side of the equation, must be done to the other, so we will add 4 from each side, x = 14.

We can check our answer by substituting our answer for the variable in the original equation.

x – 4 = 10	original equation
14 – 4 = 10	substitute 14 for x
10 = 10	answer is correct

Practice

1. a - 20 = 5

2. x - 3 = -1

3. x - 10 = 25

Answers

1. 25

2. 2

3. 35

ge249_ch1_11

PROPERTIES OF EQUALITY

SUBTRACTION PROPERTY OF EQUALITY

Now that we have covered the addition property of equality, let's move on to the subtraction property of equality.

In the equation below,

a + 8 = 50

a is the variable, or the unknown value. We must use the subtraction property of equality to find the value of a.

In a + 8 = 50, we must eliminate the 8 to get the variable, a, by itself.

$$\begin{array}{l} a + 8 = 50 \\ \quad\; -8 \quad -8 \\ \hline a \qquad = 42 \end{array}$$

Using the property of equality, we can eliminate the 8 by subtracting 8. Whatever is done to one side of the equation, must be done to the other, so we will subtract 8 from each side, a = 42.

We can check our answer by substituting our answer for the variable in the original equation.

a + 8 = 10	original equation
42 +8 = 50	substitute 42 for a
50 = 50	answer is correct

Practice

1. a + 14 = 30

2. x + 27 = -20

3. y + 8 = 34

Answers

1. 16

2. - 47

3. 26

ge249_ch1_12

PROPERTIES OF EQUALITY

MULTIPLICATION PROPERTY OF EQUALITY

Now let's take a look at the multiplication property of equality.

In the equation below,

$$\frac{2}{3}x = 20$$

x is the variable, or the unknown value. We must use the Multiplication Property of Equality to find the value of x. Why the multiplication property? To eliminate a fraction, we must multiply by its inverse or reciprocal. To find the inverse of a fraction, "flip it". For example, the inverse of $\frac{1}{2}$ is $\frac{2}{1}$.

In $\frac{2}{3}x = 20$, we must eliminate the $\frac{2}{3}$ to get the variable, x, by itself. The inverse of $\frac{2}{3}$ is $\frac{3}{2}$.

Using the property of equality, we can eliminate the $\frac{2}{3}$ by multiplying by its inverse, $\frac{3}{2}$. Whatever is done to one side of the equation, must be done to the other, so we will multiply $\frac{3}{2}$ on each side, x = 30.

$$\frac{2}{3}x = 20$$

$$\frac{3}{2} * \frac{2}{3}x = 20 * \frac{3}{2}$$

$$x = \frac{60}{2} = 30$$

$$x = 30$$

We can check our answer by substituting our answer for the variable in the original equation.

$$\frac{2}{3}x = 20$$ original equation

$$\frac{2}{3} * 30 = 20$$ substitute 30 for x

$$\frac{60}{3} = 20$$

$20 = 20$ answer is correct

Practice

1. $\frac{1}{4}a = 15$
2. $\frac{7}{9}a = -3$
3. $\frac{5}{6}y = 12$

Answers

1. 60
2. $-\frac{27}{7}$
3. $\frac{72}{5} = 14\frac{2}{5}$

ge249_ch1_13

PROPERTIES OF EQUALITY

DIVISION PROPERTY OF EQUALITY

The last property is the division property of equality.

In the equation below,

$5x = 20$

x is the variable, or the unknown value. We must use the Division Property of Equality to find the value of x.

In $5x = 20$, we must eliminate the 5.

$5x = 20$

$$\frac{5x}{5} = \frac{20}{5}$$

$x = 4$

Using the property of equality, we can eliminate the 5 by dividing by 5. Whatever is done to one side of the equation, must be done to the other, so we will divide each side by 5, $x = 4$.

We can check our answer by substituting our answer for the variable in the original equation.

$5x = 20$	original equation
$5(4) = 20$	substitute 4 for x
$20 = 20$	answer is correct

Practice

1. $10y = 100$

2. $-7a = 49$

3. $15x = 30$

Answers

1. 10

2. -7

3. 2

ge249_ch1_14

TWO STEP EQUATIONS

SOLVING TWO STEP EQUATIONS

Now it's time to put all of your Algebra skills to use. Up to this point we have been working with one step equations. Let's move on to two step equations.

Here are some examples:

$5x + 3 = 10$

$5x + 3 = 10$ Original equation

$\quad -3 \quad -3$ Subtraction Property of Equality

$5x = 7$

$\frac{5x}{5} = \frac{7}{5}$ Division Property of Equality

$x = \frac{7}{5}$

$3a + 6a - 2a = 20$

$3a + 6a - 2a = 20$ Original equation

$7a = 20$ Combine Like Terms

$\frac{7a}{7} = \frac{20}{7}$ Division Property of Equality

$a = \frac{20}{7} = 2\frac{6}{7}$

$6(a+3) - 10a = 45$

$6(a+3) - 10a = 45$ Original equation

$6a + 18 - 10a = 45$ Combine Like Terms

$-4a + 18 = 45$

$\quad -18 \quad -18$ Subtraction Property of Equality

$-4a = 27$

$\frac{-4a}{-4} = \frac{27}{-4}$ Division Property of Equality

$a = -\frac{23}{4} = -5\frac{3}{4}$

Practice

1. $15y + 7 = 37$

2. $10a - 12a - 4a = 18$

3. $3(x+8) - 14x = 22$

Answers

1. 2

2. -3

3. $\frac{2}{11}$

ge249_ch1_15

INEQUALITIES

SOLVING LINEAR INEQUALITIES

Solving inequalities are similar to the equations that we have covered, except there are inequalities in the middle, instead of an equal sign. We must still isolate the variable by eliminating all numbers by the variable.

First, let's review the inequality symbols:

>	Greater than
<	Less Than
≥	Greater Than or Equal To
≤	Less Than or Equal To

Example:

$4x - 5 \geq 11$ Original Equation

$4x - 5 + 5 \geq 11 + 5$ Eliminate the 5 by adding 5 to each side

$4x \geq 16$

$\frac{4x}{4} \geq \frac{16}{4}$ Divide by 4 on each side to eliminate the 4

$x \geq 4$

There is one exception when solving inequalities. When dividing by a negative number, the inequality sign "flips", or becomes the opposite sign. See below:

$-5x \geq 10$ Original Equation

$\frac{-5x}{-5} \geq \frac{10}{-5}$ Eliminate the -5 by dividing by -5 on each side

$x \leq -2$ Flip the inequality sign. The original was greater than or equal to, now it has been changed to less than or equal to

Practice

1. $8x - 4 \geq 60$

2. $-3x + 10 < -11$

3. $10y - 9 \geq 21$

Answers

1. $x \geq 8$

2. $x > 7$

3. $y \geq 2$

ge249_ch1_16

INEQUALITIES

GRAPHING LINEAR INEQUALITIES

Graphing an inequality involves placing a dot on a number line, then indicating whether the value is greater or less than the number given.

Let's review the inequality symbols one more time.

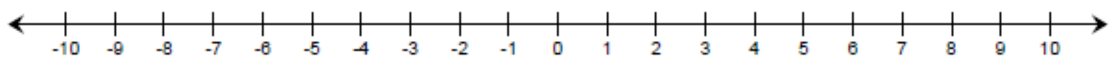
$>$ Greater than

$<$ Less Than

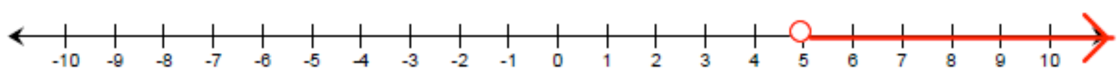
$\geq$ Greater Than or Equal To

$\leq$ Less Than or Equal To

A number line starts with zero in the middle. Positive numbers are to the right of the zero, and negative numbers are to the left.

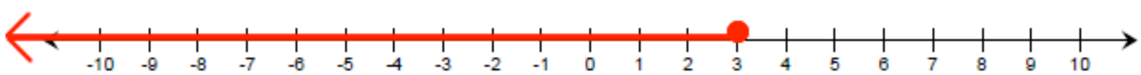


Here is an example of $x > 5$ graphed. This is read "x is greater than 5".



This graph indicates that the number is greater than 5. This is graphed with an open circle, and the arrow points in the right direction, signifying greater than.

Now let's look at $x \leq 3$ graphed. This is read "x is less than or equal to 3".



This graph indicates that the number is less than or equal to 3. This is graphed with a closed dot, and the arrow points in the left direction, signifying less than.

Here are a couple of rules to follow when graphing inequalities on a number line:

If it is……	**Then…**
> or <	Graph with an open circle
≥ or ≤	Graph with a closed dot
> or ≥	The arrow will point to the right
< or ≤	The arrow will point to the left

Practice

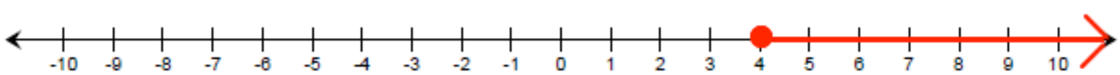
1. $x \geq 4$

2. $x < -3$

3. $x \geq -9$

Answers

1.



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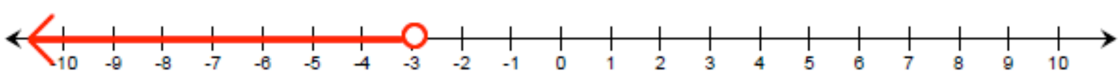

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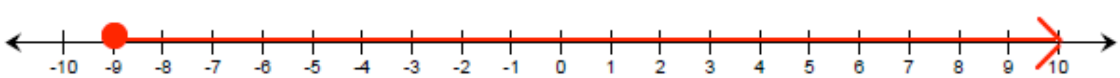

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WORD PROBLEMS WITH LINEAR EQUATIONS

SOLVING WORD PROBLEMS WITH LINEAR EQUATIONS

We can solve word problems using equations. The first step is to write equations from the problem. Specific words can be translated into math symbols.

Word	**Math Symbol**
total	=
and	+
more	+
times	*

Now let's go through a few steps in solving word problems with equations, then we will apply each step to a problem.

1. Read through the problem
2. Define your variables.
3. Write equations based on the problem.
4. Solve the equation.

Here is an example:

A local hamburger shop sold a total of 520 hamburgers and cheeseburgers Tuesday. There were 70 more cheeseburgers sold than hamburgers. How many hamburgers were sold on Tuesday?

1. Read through the problem.

2. Define the variables.

H = hamburger
C = cheeseburger

3. Write equations based on the problems. Let's look at the first sentence.

A local hamburger shop sold a total of 520 hamburgers and cheeseburgers Tuesday.

The equation from this sentence would be
H + C = 520 Hamburgers plus cheeseburgers equals 520.

The next sentence –

There were 70 more cheeseburgers sold than hamburgers.

C + 70 = H The amount of cheeseburgers, plus 70, equals the amount of hamburgers.

4. Solve the equation.

We are looking for how many hamburgers were sold.

Our equations are

H + C = 520
C + 70 = H

Let’s substitute C + 70 for H in the first equation.

$H + C = 520$	Hamburgers plus cheeseburgers equals 520.
$C + 70 + C = 520$	Substitute C + 70 for H.
$2C + 70 = 520$	Combine like terms.
$2C + 70 - 70 = 520 - 70$	To eliminate 70, subtract 70 from each side.
$2C = 450$	
$\frac{2C}{2} = \frac{450}{2}$	Divide by 2 on each side.
$C = 225$	There are 225 cheeseburgers.

From this problem, we know that 225 cheeseburgers have been sold. We want to know how many hamburgers have been sold. From the original word problem, there were 70 more hamburgers sold than cheeseburgers.

$$C + 70 = H$$
$$225 + 70 = H$$
$$\mathbf{295 = H}$$

295 hamburgers were sold.

Practice

1. A washer and dryer cost $750. The price of the washer is two times the price of a dryer. What is the cost of the dryer?

2. A local hamburger shop sold a total of 400 hamburgers and cheeseburgers Friday. There were 40 less cheeseburgers sold than hamburgers. How many cheeseburgers were sold on Friday?

3. A television and stereo cost $600. The price of the television is three times the price of a stereo. What is the cost of the stereo?

Answers

1. 250

2. 220

3. 150

ge249_ch1_18

SUMMARY

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In summary, you learned how to add, subtract, multiply and divide with negative numbers. You also learned how to evaluate exponents, apply substitution, apply the distributive property, and combine like terms. Finally you learned how to solve equations with the Properties of Equality, solve and graph inequalities, and solve word problems with linear equations.